# Mathematical Model for a Standing-Wave Acoustic Parametric Source (SWAPS)

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## ABSTRACT

A standing-wave acoustic parametric source (SWAPS) is proposed as a small, low-frequency underwater sound source. SWAPS is a liquid-filled cylindrical tube which is driven at one end by a piston transducer and terminated on the other end by a pressure-release reflector to form a resonant cavity. The piston is driven simultaneously at two high frequencies $\omega_1$ and $\omega_2$ which are at or near resonance for the plane-wave mode in the cavity. The resulting large-amplitude primary sound waves mix nonlinearly to produce secondary sound waves. The secondary wave at the low difference frequency $\omega_1 - \omega_2$ passes easily through the relatively thin tube walls and radiates to the far field. In this report the far-field radiation at the difference frequency is calculated by use of the virtual-source-density method. Two special cases are discussed: in the first the high frequencies are neighboring resonance frequencies so that the length of SWAPS is nearly 1/2 wavelength at the difference frequency in the liquid in the tube; in the second both high frequencies lie within the bandwidth of a single resonance of the cavity; in this second case SWAPS can be as short as 1/4 wavelength at the high frequency. The results are expressed in terms of a quality factor that describes the resonance properties of the system when difference-frequency radiation is being generated. This quality factor, which must be determined experimentally, is an input to the mathematical model. Comparison is made to a traveling-wave acoustic parametric source which employs the same piston transducer as a pump.

## PROBLEM STATUS

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# MATHEMATICAL MODEL FOR A STANDING-WAVE ACOUSTIC PARAMETRIC SOURCE (SWAPS)

## INTRODUCTION

Within the past decade the generation of narrow-beam, low-frequency acoustic radiation from a relatively small piston source has been accomplished by the use of the traveling-wave acoustic parametric source. Here the piston source is driven simultaneously by two primary signals of high frequency $\omega_1$ and $\omega_2$. During subsequent propagation a low-frequency secondary wave at the difference frequency $\omega_1 - \omega_2$ is generated by the nonlinear interaction of the primary sound waves in the water medium. This so-called endfire array, first proposed by Westervelt [1] and confirmed experimentally by Bellin and Beyer [2], has been the subject of extensive theoretical and experimental investigation by Berktay [3] and others [4-6]. The desirability of such a device is enhanced by the lack of sidelobes in the radiation pattern and by the relatively large bandwidth at the difference frequency. Its chief disadvantage is the low efficiency of the conversion due to the absorption and diffraction of the primary waves and the nonlinear generation and subsequent absorption of both the harmonics of the primary waves and the sum frequency $\omega_1 + \omega_2$. This limitation is unlikely to be eliminated by future work.

In 1965 Dunn, Kuljis, and Welsby [7] briefly examined the generation of a 47-kHz sound from two primary waves at 326 kHz and 373 kHz in a spherical-standing-wave system. However, they were primarily concerned with subharmonic generation when cavitation is present and did not pursue the matter further.

In this report we describe a low-frequency sound source called a standing-wave acoustic parametric source (SWAPS) which increases the nonlinear conversion by "folding" the traveling wave to produce a standing wave [8]. This is accomplished by use of a closed cylindrical tube that is near resonance for both primary sound waves. Because the parametric effect is inherently nonlinear, an increased generation of sound at the difference frequency results. This difference frequency passes easily through the relatively thin tube walls and ends and radiates to the far field. We calculate the far-field radiation from SWAPS by use of the virtual-source-density method used by Westervelt [1]. Included in our discussion is a comparison to a traveling-wave acoustic parametric source which employs the same piston transducer to generate the primary waves.

## DESCRIPTION OF SWAPS

SWAPS is envisioned to be a liquid-filled circular cylindrical tube or cavity of inner radius $a$ and length $\ell$ with a piston transducer mounted on one end and a pressure-release reflector mounted on the other end (Fig. 1). The piston is driven simultaneously at the two primary frequencies $\omega_1$ and $\omega_2$. The walls are made sufficiently rigid to allow predominantly plane-wave propagation of the primaries. Both $\omega_1$ and $\omega_2$ are chosen to be near a resonance

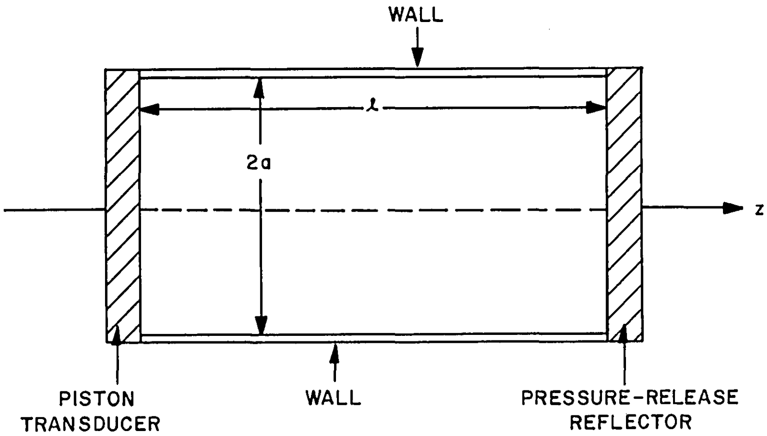


Fig. 1 — Standing-wave acoustic parametric source

frequency of the plane-wave mode of the cavity. Two special cases are considered: (1) $\omega_1$ and $\omega_2$ are neighboring resonance frequencies; (2) $\omega_1$ is close enough to $\omega_2$ so that they both lie well within the bandwidth of a single resonance frequency. The resulting primary waves present in this standing-wave system now interact nonlinearly to produce sum and difference components as well as various harmonic components. The difference frequency $\omega_d = \omega_1 - \omega_2$ is chosen to be much smaller than either primary frequency. Thus the relatively thin walls and ends of the tube can also be made nearly transparent acoustically at the difference frequency, so that the difference frequency radiates unimpeded to the far field.

The use of a pressure-release reflector should inhibit the growth of both the sum frequency and harmonics of the primaries. This occurs because of the 180° phase shift that each frequency component in a plane wave undergoes upon reflection from a pressure-release boundary. Consider a sinusoidal wave leaving the piston. As it propagates, harmonics are generated with a fixed phase relationship to the fundamental. Ignoring dispersion, this phase relationship is maintained while the "most stable" waveform, the sawtooth, is approached. When the distorted wave is reflected from the pressure release end, the 180° phase shift produces a "least stable" waveform or reverse sawtooth. During subsequent propagation back to the piston, new harmonic generation cancels the existing harmonic content, and the waveform tends to return to a sinusoid. The sum frequency $\omega_s = \omega_1 + \omega_2$ will be close in frequency to the second harmonic of both primaries and will be inhibited in a similar manner. In addition, any dispersion that exists in the tube will tend to inhibit the growth of harmonics and the sum frequency, since it will also tend to destroy the stable phase relationship between the various components. Thus competing nonlinear interactions are inhibited, and more primary energy remains available for generation of the difference frequency.

The liquid used in SWAPS can be chosen to optimize the performance. For example an ideal liquid might have a characteristic acoustic impedance equal to that of the surrounding water medium so that the difference frequency is transmitted effectively into the water, have a high degree of nonlinearity to enhance the interaction of the primaries, have small attenuation of the primaries due to linear loss mechanisms such as viscous absorption in order to enhance the resonance properties of the cavity, have a high cavitation threshold (if cavitation proves to be undesirable), and have a low sound speed to maximize the acoustic size of the interaction volume.

## MATHEMATICAL MODEL OF SWAPS

The basic equations of acoustics describing the conservation of mass, momentum, and energy and the equation of state of the medium are inherently nonlinear. Under the assumptions of infinitesimal wave motion, the nonlinear terms can be neglected, and the linear wave equation is obtained. Fortunately, for most applications, infinitesimal theory is quite adequate. However, when the wave motion is of large amplitude so that the acoustic particle velocity is not negligible when compared with the sound speed, the nonlinear terms must be retained. It is the existence of these nonlinear terms that leads to mixing of the primary waves to produce secondary waves which consist of harmonics of each primary frequency component and intermodulation components such as sum and difference frequencies. References 9 through 12 give detailed discussions of nonlinear acoustics.

Because of the intractable nature of the nonlinear equations one usually makes a number of simplifying assumptions or idealizations to obtain a solution. For example, if one assumes that the secondary waves are small compared with the primary waves, first-order perturbation theory can be used. Such an approach was used by Westervelt [1] in describing the parametric acoustic array, which we call a traveling-wave acoustic parametric source and apply the acronym TWAPS.

Westervelt used as his starting point the exact equation for arbitrary fluid motion derived by Lighthill [13]. Using perturbation theory and neglecting viscosity, he derived the inhomogeneous wave equation

$$\square^2 P = -\rho_0 \frac{\partial q}{\partial t} \equiv -f, \tag{1}$$

where $P$ is the acoustic pressure of the secondary wave which includes the harmonic and intermodulation components and $\rho_0$ is the equilibrium mass density. The virtual-simple-source-strength density $q$ which results from the primary-wave pressure $P_i$ is given by

$$q = \frac{\Gamma}{\rho_0^2 c_0^4} \frac{\partial}{\partial t} (P_i^2), \tag{2}$$

where $c_0$ is the sound speed for infinitesimal waves. The parameter $\Gamma$ is defined by

$$\Gamma = 1 + \frac{\rho_0}{2c_0^2} \left(\frac{\partial^2 P}{\partial \rho^2}\right)_{\rho = \rho_0} \tag{3}$$

or

$$\Gamma = 1 + \frac{B}{2A}, \tag{4}$$

where the nonlinearity parameter B/A is a measure of the nonlinear response of the liquid medium. For water at 20°, B/A is about 5. Other liquids have values of B/A that lie between about 4 and 11 [11].

Westervelt assumed a primary wave consisting of two collimated collinear plane waves of frequencies $\omega_1$ and $\omega_2$ emanating from a common source. He reintroduced the effects of viscosity in an ad hoc way by assuming that the primaries are attenuated by the usual absorption coefficient of linear acoustics. Substituting this primary wave into Eq. (2) led to a virtual-source density with the frequency components $2\omega_1$, $2\omega_2$, $\omega_s$, and $\omega_d$. He then obtained the far-field radiation at the difference frequency by using the $\omega_d$ virtual-source-density term to obtain the inhomogeneous term $f$ in Eq. (1) and by integrating this expression times the free-space Green's function over the cylindrical volume containing the primary waves. He assumed that the difference frequency is low enough so that its attenuation is negligible. This model assumes that the transfer of energy from the primaries into secondary waves such as harmonics and sum and difference components is small enough so that nonlinear attenuation of the primaries is negligible. Recently attempts have been made to modify the primary-wave distribution to approximate the nonlinear attenuation due to harmonic generation [6]. At large primary amplitudes (or at very low primary frequencies), this loss mechanism will dominate linear absorption.

We now calculate the difference frequency radiated from SWAPS into the far field by the perturbation approach of Westervelt. First we obtain the primary-wave distribution. The piston end of SWAPS is assumed to be vibrating with a normal velocity $V$ given by

$$V = V_1 \sin \omega_1 t + V_2 \sin \omega_2 t. \tag{5}$$

The axis of the tube is taken to be the $z$ axis with the piston at $z = 0$. The reflector end of the tube is pressure released, so that

$$P_i = 0 \text{ at } z = \ell. \tag{6}$$

The walls of the tube are assumed rigid at $\omega_1$ and $\omega_2$, and plane wave motion is assumed. The linear attenuation coefficients for the primaries in the tube $\alpha_1$ and $\alpha_2$ include contributions from such linear loss mechanisms as boundary-layer and mainstream-viscous attenuation. The solution to the one-dimensional linear wave equation subject to the boundary conditions given in Eqs. (5) and (6) is given by

$$\begin{aligned} P_i &= P_1 + P_2 \\ &= \rho_0 V_1 Re \left\{ \frac{\omega_1 \sin [(\ell - z)(k_1 - i\alpha_1)] e^{i\omega_1 t}}{(k_1 - i\alpha_1) \cos [\ell(k_1 - i\alpha_1)]} \right\} \\ &\quad + \text{a similar term for } P_2, \end{aligned} \tag{7}$$

where $Re$ indicates the real part of the expression in braces and $k$ is the wavenumber.

We now assume that the linear attenuation is small, so that

$$\alpha_1 \ell << 1, \tag{8}$$

and

$$\alpha_2 \ell << 1. \tag{8}$$

In addition we assume that the length $\ell$ is such that both $\omega_1$ and $\omega_2$ are equal to or nearly equal to a resonance frequency $\omega_n$ of the standing-wave system, that is,

$$\omega_1 - \omega_n << \omega_n$$

and (9)

$$\omega_n - \omega_2 << \omega_n,$$

where it is assumed that $\omega_1$ and $\omega_2$ are located on opposite sides of $\omega_n$. When the small dispersion and attenuation that will exist in the tube are neglected, the resonance frequencies $\omega_n$ are given by

$$\omega_n = \frac{(2n + 1)\pi c_0}{2\ell}, \tag{10}$$

where the integer $n$ indicates the order of the resonance and $c_0$ is the free-field sound speed for the liquid in SWAPS.

With these assumptions the linear solution can be written in the simplified form

$$P_i = \rho_0 c_0 [V_1 A_1 \sin(\omega_1 t - \phi_1) \cos k_1 z + V_2 A_2 \sin(\omega_2 t - \phi_2) \cos k_2 z], \tag{11}$$

where the amplitude and phase factors are given by

$$A_1 = \frac{1}{\alpha_1 \ell \{1 + [(\omega_1 - \omega_n)/(\alpha_1 c_0)]^2\}^{1/2}}$$

and (12)

$$\phi_1 = \tan^{-1}[(\omega_1 - \omega_n)/(\alpha_1 c_0)],$$

and similarly for $A_2$ and $\phi_2$. At resonance, $\omega_1 = \omega_n$ and $A_1$ and $A_2$ have their maximum values of $1/\alpha_1 \ell$ and $1/\alpha_2 \ell$ respectively. From Eqs. (12) we see that the conditions that $\omega_1$ and $\omega_2$ lie within the bandwidth of the resonance at $\omega_n$ can be expressed as

$$\frac{\omega_1 - \omega_n}{\alpha_1 c_0} \leqslant 1$$

and (13)

$$\frac{\omega_n - \omega_2}{\alpha_2 c_0} \leqslant 1.$$

If the amplitudes of the primary waves were small and if the assumed boundary conditions at the walls and ends were met, Eq. (11) would be a good representation of the primary waves. However these primary waves would not produce much difference-frequency radiation. Therefore, if significant difference-frequency radiation is desired, then the amplitudes of the primaries must be large and the infinitesimal theory used to obtain Eq. (11) is no longer valid. Even if the pressure-release reflector works perfectly, the harmonics of the primaries as well as other high-frequency intermodulation components cannot now be treated as small secondary waves which have a negligible effect on the primary waves and can be calculated from first-order perturbation theory. Thus we modify the primary-wave distribution to include these high-frequency components and still consider the difference frequency $\omega_d$ to be the secondary wave. Since we are interested in only $\omega_d$, we need to know only the distribution of the fundamental components of the primaries. We assume that this distribution is given by

$$P_i = \rho_0 c_0 [V_1 Q_1 \sin(\omega_1 t - \phi_1) \cos k_1 z + V_2 Q_2 \sin(\omega_2 t - \phi_2) \cos k_2 z], \tag{14}$$

where the quality factors $Q_1$ and $Q_2$ contain the dependence given by Eqs. (12) and an additional unspecified factor representing the loss of energy from the fundamental components into the generation of high-frequency intermodulation components. We also include in $Q_1$ and $Q_2$ the loss of energy from $\omega_1$ and $\omega_2$ due to generation of the difference frequency itself. This apparent departure from second-order perturbation theory is justified by the fact that the difference frequency does not build up in the standing-wave system but instead radiates to the far field. Consequently the amount of difference frequency present in the region occupied by the primaries can be small compared with the primaries even if a significant amount of difference frequency is radiated. Therefore $Q_1$ and $Q_2$ determine the amplitudes of the primaries during difference-frequency generation and are each a function of both piston velocity components $V_1$ and $V_2$. To simplify the analysis, we assume that $V_1$ is equal to $V_2$ and that $\omega_1 - \omega_n = \omega_n - \omega_2 = \omega_d/2$ when both primaries lie within a single resonance. Then since $\omega_1 \approx \omega_2 \equiv \omega$, so that $\alpha_1 \approx \alpha_2 \equiv \alpha$, we have $A_1 \approx A_2$, $Q_1 \approx Q_2 \equiv Q$, and $\phi_1 = -\phi_2 \equiv \phi_0$. The primary distribution becomes

$$P_i = \rho_0 c_0 VQ [\sin(\omega_1 t - \phi_0) \cos k_1 z + \sin(\omega_2 t + \phi_0) \cos k_2 z]. \tag{15}$$

Thus we require that the behavior of $Q$ as a function of $V$ be determined experimentally as an input to this mathematical model.

Substituting the expression for $P_i$ into Eqs. (1) and (2) and retaining only the terms that contribute to the difference frequency, we have

$$\begin{aligned} f = \rho_0 \frac{\partial q}{\partial t} = -\frac{\Gamma \rho_0 V^2 Q^2 k_d^2}{4} &[\cos(\omega_d t - k_d z - 2\phi_0) + \cos(\omega_d t + k_d z - 2\phi_0) \\ &+ \cos(\omega_d t - k_s z - 2\phi_0) + \cos(\omega_d t + k_s z - 2\phi_0)]. \end{aligned} \tag{16}$$

It can be shown that the last two terms do not contribute significantly to the far field when $\omega_s >> \omega_d$ and $k_s >> k_d$. The difference-frequency pressure at the field point $\vec{R}$ is given by

$$P_d(\vec{R}) = \frac{1}{4\pi}\int \frac{f e^{-ik_d|\vec{R} - \vec{r}|}}{|\vec{R} - \vec{r}|} dV, \tag{17}$$

where the integration is over the circular cylindrical volume of radius $a$ and length $\ell$ and where $\vec{r}$ is the source point (Fig. 2). We assume for the present that SWAPS is filled with water (or whatever the surrounding liquid is). We take $\vec{R}$ to be in the far field and use the asymptotic form for the free-space Green's function. The free-space Green's function is appropriate for the difference frequency, since we assumed that the walls and ends of SWAPS are acoustically transparent at $\omega_d$. Neglecting the last two terms in Eq. (16), converting to complex notation, and choosing cylindrical coordinates $(\sigma, \beta, z)$ for the source point and spherical coordinates $(R, \theta, \phi)$ for the field point, we obtain the expression

$$P_d(R, \theta) = SF(\theta)\,\frac{e^{i(\omega_d t - k_d R - 2\phi_0)}}{R}, \tag{18}$$

with

$$SF(\theta) = -\frac{\Gamma \rho_0 V^2 Q^2 k_d^2}{16\pi}\int_0^a \int_0^{2\pi} \int_0^{\ell} e^{ik_d[\sigma\sin\theta\cos(\phi-\beta)+z\cos\theta]}\left(e^{-ik_d z} + e^{ik_d z}\right)\,\sigma\, d\sigma d\beta dz, \tag{19}$$

where $S$ is the source level and $F(\theta)$ is the far-field pressure distribution. The integration over $\beta$ of the factor depending on $\beta$ and $\sigma$ produces $2\pi J_0(k_d\sigma \sin\theta)$, where $J_0$ is the zero-order Bessel function, and subsequent integration over $\sigma$ produces the aperture factor

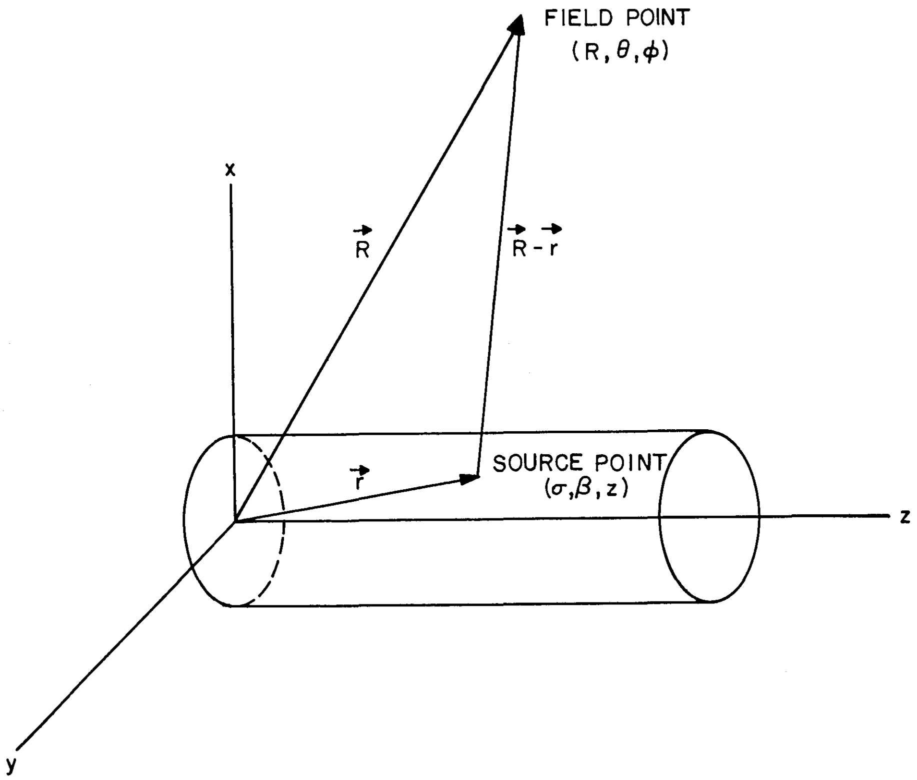


Fig. 2 — Geometry of the problem

$$H = \frac{2\pi a}{k_d \sin\theta} J_1 (k_d a \sin\theta). \tag{20}$$

We usually choose $k_d a \sin\theta \ll 1$, so that $J_1$ can be replaced by its lowest order term and $H$ has its maximum value

$$H \approx \pi a^2. \tag{21}$$

Integration over $z$ of the remaining factors produces two oppositely directed end-fired-array terms that are $k_d \ell$ out of phase with each other:

$$E(\theta) = e^{ik_d \ell} \left\{ \frac{\sin [k_d \ell \cos^2 (\theta/2)]}{k_d \ell \cos^2(\theta/2)} \right\} + \left\{ \frac{\sin [k_d \ell \sin^2(\theta/2)]}{k_d \ell \sin^2(\theta/2)} \right\}. \tag{22}$$

The second term in braces can be obtained from the first term in braces by replacing $\theta$ with $180° - \theta$. Collecting terms, we obtain the source level

$$S = \Gamma \rho_0 V^2 Q^2 k_d^2 a^2 \ell |E(0)|/16 \tag{23}$$

and the far-field pressure distribution

$$|F(\theta)| = \frac{J_1 (k_d a \sin\theta)}{(1/2) k_d a \sin\theta} |E(0)|^{-1} \frac{1}{k_d \ell} \left[ \frac{\sin^2 [k_d \ell \cos^2(\theta/2)]}{\cos^4(\theta/2)} + \frac{\sin^2 [k_d \ell \sin^2(\theta/2)]}{\sin^4(\theta/2)} + 2 \cos k_d \ell \, \frac{\sin [k_d \ell \sin^2(\theta/2)] \sin [k_d \ell \cos^2(\theta/2)]}{\sin^2(\theta/2) \cos^2(\theta/2)} \right]^{1/2}, \tag{24}$$

where

$$|E(0)| = \left[ \frac{\sin^2 k_d \ell}{k_d^2 \ell^2} + \frac{\sin 2k_d \ell}{k_d \ell} + 1 \right]^{1/2} \tag{25}$$

and where $|F(\theta)|$ is normalized to unity in the forward direction ($\theta = 0$).

In the first special case, where $\omega_1$ and $\omega_2$ are two neighboring resonance frequencies, it can easily be shown that $k_d \ell = \pi$, so that $\ell = \lambda_d/2$. Then $|E(0)|$ is equal to unity, and the source level and far-field pressure distribution for this half-wavelength SWAPS (HWS) are given by

$$S_{HWS} = \pi \Gamma \rho_0 V^2 Q^2 k_d a^2/16 \tag{26}$$

and

$$|F(\theta)| = \left[ \frac{J_1 (k_d a \sin\theta)}{(1/2) k_d a \sin\theta} \right] \left[ \frac{\cos\theta \cos (\pi/2 \cos\theta)}{(\pi/4) \sin^2\theta} \right]. \tag{27}$$

We assume that the first factor is nearly equal to unity and plot the second factor in Fig. 3. The far-field pressure distribution becomes zero in the broadside direction, similar to the far-field pressure distribution of a linear dipole. The 3-dB-down points occur at $\theta = 53^\circ$ and $\theta = 127^\circ$. Complete nulls occur in this pattern because the two oppositely directed waves in the virtual-source density produce complete cancellation over a half wavelength. If SWAPS contained a liquid with a sound speed less than that of water, then the length of the tube would be less than a half wavelength in water and the cancellation would be incomplete. This liquid should also have a density that is correspondingly larger than that of water, so that the characteristic impedances would be nearly equal and the effect of the boundary between the two liquids would be minimized. This impedance match coupled with the small acoustical size of SWAPS at the difference frequency should allow the far field to be calculated as if SWAPS were filled with water insofar as the phase terms are concerned. Of course the quantities $\rho_0$, $c_0$, and $\Gamma$ appearing in the amplitude of $f$ should be evaluated for the actual liquid in SWAPS. For example, if the liquid in SWAPS were similar to carbon tetrachloride with a sound speed of $10^5$ centimeters per second and a density of 1.5 grams per cubic centimeter, the far-field pressure distribution calculated using Eq. (24) would be nearly omnidirectional with only 1/2 dB difference between the maxima at 0° and 180° and the minima at 90° and 270°.

The total power $W_{HWS}$ radiated at the difference frequency by the half-wavelength SWAPS is obtained by numerical integration of $|F(\theta)|^2$ with Gaussian quadrature. The result is

$$W_{HWS} = \frac{1.64\pi S_{HWS}^2}{2\rho_0 c_0}. \tag{28}$$

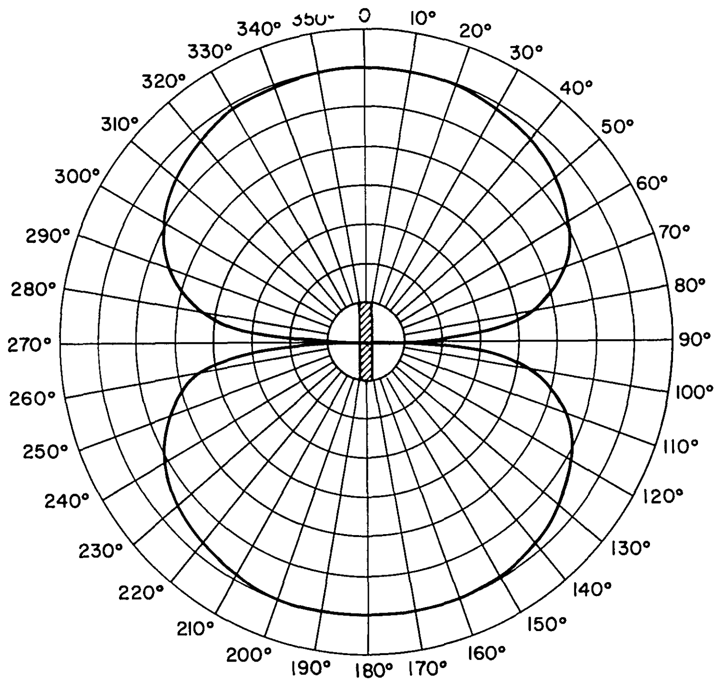


Fig. 3 — Far-field pressure distribution for the half-wavelength SWAPS

Of perhaps more interest is the second special case, where both $\omega_1$ and $\omega_2$ lie within the bandwidth of a single resonance of the standing-wave system. In this case the length of SWAPS is nearly equal to

$$\ell = \left(\frac{2m + 1}{4}\right) \lambda, \tag{29}$$

where $m$ is an integer and $\lambda$ is the wavelength in SWAPS at either primary frequency. Consequently $\ell$ can be considerably shorter than a half wavelength at the difference frequency to take advantage of the fact that $S$ depends on $Q^2\ell$. Thus if $Q^2$ increases faster than $1/\ell$ as $\ell$ is decreased to values corresponding to smaller integer values for $m$, there will be an overall increase in $S$. For small-amplitude waves it is seen from Eq. (12) that $Q^2\ell$ increases nearly as $1/\ell$. For large-amplitude waves, $Q^2\ell$ should also increase as the length is decreased from that of a half-wavelength SWAPS. At some optimum length, however, increased nonlinear losses, possible cavitation, and heating effects will cause $Q^2\ell$ to begin to decrease.

For this short SWAPS ($SS$), the far-field pressure distribution predicted by Eq. (24) is omnidirectional, and the source level and radiated power are given by

$$S_{SS} = \Gamma \rho_0 V^2 Q^2 k_d^2 a^2 \ell/8, \tag{30}$$

and

$$W_{SS} = \frac{4\pi S_{SS}^2}{2\rho_0 c_0}. \tag{31}$$

To get some feeling for these expressions, we compare the results for the short SWAPS to those obtained by Westervelt for TWAPS, although we recognize that SWAPS and TWAPS are designed for different applications. For simplicity we assume that TWAPS is operating with the same piston velocity as SWAPS. The source level and radiated power calculated by Westervelt are designated by $S_T$ and $W_T$, and we use $\alpha'$ to represent the free-field attenuation of the primaries in seawater. This attenuation coefficient should include contributions from viscous attenuation, the generation of harmonics and other intermodulation components, and diffraction. For the case where the short SWAPS is filled with water, we have

$$\frac{S_{SS}}{S_T} = Q^2 \alpha' \ell \tag{32}$$

and

$$\frac{W_{SS}}{W_T} = 2Q^4 \alpha' \ell k_d \ell/\pi. \tag{33}$$

Thus the achievement of a source level comparable to that of TWAPS requires that $Q^2$ be comparable to $1/\alpha'\ell$. We are not predicting values for $Q$ at the present time and emphasize again that it is a parameter that must be experimentally determined as an input to this mathematical model. The ratio of total radiated powers is much more favorable than the ratio of source levels, since $k_d$ is expected to be much larger than $\alpha'$, so that

$$\frac{W_{SS}}{W_T} >> \left(\frac{S_{SS}}{S_T}\right)^2. \tag{33}$$

## SUMMARY


We summarize some of the more important features of the standing-wave acoustic parametric source (SWAPS):

- SWAPS is designed to be a small, low-frequency underwater sound source.
- Two special modes of operation are available:

—The two primary frequencies are neighboring resonance frequencies for the standing-wave system in SWAPS. In this case the length of SWAPS is a half wavelength at the difference frequency in the liquid in SWAPS.

—The two primary frequencies are both within the bandwidth of a single resonance frequency for the standing-wave system in SWAPS. In this mode the length of SWAPS is nearly equal to an odd number of quarter wavelengths at either primary frequency in the liquid in SWAPS and thus can be considerably shorter than the half-wavelength SWAPS.

- SWAPS is expected to be a more effective radiator of difference frequency than a traveling-wave acoustic parametric source (TWAPS) because (a) high-energy density and, consequently, increased nonlinear conversion is achieved by simultaneous resonance or near resonance of the two primary signals and (b) competing nonlinear interactions such as the generation of harmonics of the primaries and the generation of the sum frequency are inhibited by the pressure-release reflector.

- Since SWAPS is short acoustically at the difference frequency, it will not have the high directivity associated with TWAPS. Instead SWAPS is intended for applications that require an omnidirectional or perhaps a dipolar far-field pressure distribution. If directivity is desired, an array of SWAPS sources can be used. The frequently encountered acoustic interaction problem inherent with an array of small conventional sources should not occur here because the piston which produces the high-frequency primary waves in a SWAPS source will not be affected appreciably by the low-frequency difference-frequency radiation from neighboring SWAPS sources. The high-frequency primary waves are contained within the SWAPS sources and therefore cannot affect the piston in a neighboring SWAPS source.

- The liquid used in SWAPS can be chosen to optimize the performance. Some important parameters to consider are sound speed, density, nonlinearity, attenuation coefficient, and cavitation strength.


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## REFERENCES

1. P.J. Westervelt, "Parametric Acoustic Array," J. Acoust. Soc. Amer. 35, 535-537 (1963).
2. J.L.S. Bellin, and R.T. Beyer, "Experimental Investigation of an End-Fire Array," J. Acoust. Soc. Amer. 34, 1051-1054 (1962).
3. H.O. Berktay, "Possible Exploitation of Non-Linear Acoustics in Underwater Transmitting Applications," J. Sound Vib. 2, 435-461 (1965).
4. R.H. Mellen, W.L. Konrad, and D.G. Browning, "Approximate Scaling Laws for Parametric Sonar Transmitter Design," Proceedings of the British Acoustical Society Meeting on Nonlinear Acoustics, Birmingham, England, Apr. 1971.
5. T.G. Muir, "An Analysis of the Parametric Acoustic Array for Spherical Wave Fields," Applied Research Lab., Univ. Texas, Tech. Report ARL-TR-71-1, May 1971.
6. R.H. Mellen, and M.B. Moffett, "A Model for Parametric Sonar Radiator Design," U.S. Navy Journal of Underwater Acoustics 22, 105-116 (1972).
7. D.J. Dunn, M. Kuljis, and V.G. Welsby, "Non-linear Effects in a Focused Underwater Standing Wave Acoustic System," J. Sound Vib. 2, 471-476 (1965).
8. A.L. Van Buren, and P.H. Rogers, "Mathematical Model for a Standing Wave Acoustic Parametric Source," J. Acoust. Soc. Amer. 53, 382 (A) (1973).
9. R.T. Beyer, "Nonlinear Acoustics," chapter 10 in *Physical Acoustics*, Vol. II, Part B, edited by W.P. Mason, Academic Press, New York, 1965.
10. D.T. Blackstock, "Nonlinear Acoustics (Theoretical)," chapter 3n in *American Institute of Physics Handbook*, 3rd edition, edited by D.E. Gray, McGraw-Hill, New York, 1972.
11. R.T. Beyer, "Nonlinear Acoustics (Experimental)," chapter 3o in *American Institute of Physics Handbook*, 3rd edition, edited by D.E. Gray, McGraw-Hill, New York, 1972.
12. L.D. Rozenberg, *High-Intensity Ultrasonic Fields*, Plenum Press, New York, 1971.
13. M.J. Lighthill, Math. Revs. 19, 915 (1958).

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13. ABSTRACT


A standing-wave acoustic parametric source (SWAPS) is proposed as a small, low-frequency underwater sound source. SWAPS is a liquid-filled cylindrical tube which is driven at one end by a piston transducer and terminated on the other end by a pressure-release reflector to form a resonant cavity. The piston is driven simultaneously at two high frequencies $\omega_1$ and $\omega_2$ which are at or near resonance for the plane-wave mode in the cavity. The resulting large-amplitude primary sound waves mix nonlinearly to produce secondary sound waves. The secondary wave at the low difference frequency $\omega_1 - \omega_2$ passes easily through the relatively thin tube walls and radiates to the far field. In this report the far-field radiation at the difference frequency is calculated by use of the virtual-source-density method. Two special cases are discussed: in the first the high frequencies are neighboring resonance frequencies so that the length of SWAPS is nearly 1/2 wavelength at the difference frequency in the liquid in the tube; in the second both high frequencies lie within the bandwidth of a single resonance of the cavity; in this second case SWAPS can be as short as 1/4 wavelength at the high frequency. The results are expressed in terms of a quality factor that describes the resonance properties of the system when difference-frequency radiation is being generated. This quality factor, which must be determined experimentally, is an input to the mathematical model. Comparison is made to a traveling-wave acoustic parametric source which employs the same piston transducer as a pump.

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